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On Analog Simulation of Ionization Cooling of Muons

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Abstract

Analog simulation, proposed here as an alternative approach for the study of ionization cooling of muons, is a scaled cooling experiment, using protons instead of muons as simulation particles. It is intended to be an effective and flexible, quick and inexpensive experiment for the understanding and validation of unprecedentedly complicated cooling physics, for the demonstration and optimization of various elaborated techniques for beam manipulation in 6D phase space. It can be done and perhaps should be done before the costly and time-consuming development of extremely challenging, muon-specific cooling technology. In a nutshell, the idea here is to build a toy machine in a playground of ideas, before staking the Imperial Guard of Napoleon into the bloody battlefield of Waterloo.

1 INTRODUCTION

A muon collider [1] is possible only if ionization cooling works effectively in reducing the huge 6D phase space volume of muons inherent from creation. Currently, ionization cooling is investigated through three approaches: theory, digital simulation, and a demonstration experiment [2]. To understand that there is a need for yet another approach of investigation, let us conceptually divide our R&D objectives into two parts: physics and technology.

The first part includes the understanding and validation of ionization cooling physics, as well as demonstration and optimization of various elaborated techniques for beam manipulation in 6D phase space. The second part involves developing and testing specific technology and hardware required for muon cooling, such as superstrong solenoid field, lithium lens, robust liquid H_2 absorber, and high gradient acceleration structure.

Apparently, the first part has to be done first, for if it turns out to be negative, there would not be a need to carry out the second part that is bound to be costly and time-consuming. Questions then arise. Are we really confident that the study on the first part would not lead to exclusion on physics ground of the feasibility of the required cooling? Furthermore, could the study on the first part be carried out reliably and conclusively without experiment?

My responses to both questions are negative. First, we must recognize that the complexity of the problem we are dealing with is unprecedented in accelerator physics, when taking into account the reality of non-paraxial beam manipulation, strong nonlinearity, and possibly space charge effects. Second, theory on complicated subject is often based on simple and idealized models, and digital simulation, when integrated to be inclusive, is often too complicated to be conclusive, thus experiment is highly desirable as a benchmark and a reality check for both.

Instead of advocating a full-fledged demonstration experiment [2], which is a major development of muon-specific technology, for physics validation, we propose an alternative approach of experiment, analogy simulation, which requires little or no technology development. Specifically, analog simulation is a scaled cooling experiment, using protons instead of muons as simulation particles for easier source production, beam handling and cooling diagnostics. With proper choice of parameters, analog simulation can be designed as an effective and flexible, quick and inexpensive experiment to extract relevant physics.

Of course, proton and muon are different in numerous aspects, such as mass, lifetime, and nuclear interaction through matters, but the effects due to these differences can be scaled or normalized to a large extent in a broad sense, therefore, essential physics can still be extracted. As such, proton cooling can be used as a benchmark for the development of cooling theory and digital simulation. It can also offer insights and guidelines to optimal component and system designs for ionization cooling of muons. In the game of “scaled experiment”, what we can learn and benefit from are limited only by our own imagination.

2 BASIC COOLING THEORY

We review basic concepts and results of ionization cooling theory, with a focus on Robinson-Liouville theorem [3, 4]. This preparation is necessary for later discussion and direct comparison of cooling of protons and muons. Assuming upright ellipses, normalized 6D emittance is, $\epsilon_6 = \epsilon_x\epsilon_y\epsilon_z$, where under paraxial approximation

$$x' = \frac{dx}{dz} \ll 1, \quad y' = \frac{dy}{dz} \ll 1, \quad \delta_p = \frac{\sigma_p}{p} \ll 1,$$

normalized 2D emittances are

$$\epsilon_x = \beta\gamma\sigma_x\sigma_{x'}, \quad \epsilon_y = \beta\gamma\sigma_y\sigma_{y'}, \quad \epsilon_z = \beta\gamma\sigma_z\delta_p.$$

The fractional differentials then satisfy

$$\frac{d\epsilon_6}{\epsilon_6} = \frac{d\epsilon_x}{\epsilon_x} + \frac{d\epsilon_y}{\epsilon_y} + \frac{d\epsilon_z}{\epsilon_z}.$$

Following Palmer [5], we classify all average effects as cooling and all stochastic effects as heating

$$\frac{d\epsilon_x}{\epsilon_x} = \frac{d_c\epsilon_x}{\epsilon_x} + \frac{d_h\epsilon_x}{\epsilon_x},$$

and define partition numbers for cooling and heating by

$$J_x = \frac{d_c\epsilon_x/\epsilon_x}{dp/p}, \quad K_x = \frac{d_h\epsilon_x/\epsilon_x}{dp/p}.$$

Cooling in 2D phase space requires $J_x + K_x > 0$, since $dp < 0$ in an absorber. Likewise in 6D, we have

$$\frac{d\epsilon_6}{\epsilon_6} = (J_6 + K_6)\frac{dp}{p},$$

where

$$J_6 = J_x + J_y + J_z, \quad K_6 = K_x + K_y + K_z.$$

Cooling in 6D phase space requires $J_6 + K_6 > 0$.

Next, we derive partition numbers for each dimensions. For transverse cooling it is easy to show that $J_x = 1$ and $J_y = 1$. To find J_z, we start from an alternative expression, $\epsilon_z = c\sigma_t\sigma_\gamma$, derived with $\sigma_z = \beta c\sigma_t$, $\sigma_\gamma = \beta\sigma_\eta$, $\eta = \beta\gamma$, $d\gamma = \beta d\eta$. Since σ_t is constant, we have

$$\frac{1}{\epsilon_z}\frac{d\epsilon_z}{dz} = \frac{1}{\sigma_\gamma}\frac{d\sigma_\gamma}{dz}. \tag{1}$$

Then, using the relation [1, 5]

$$\frac{1}{\sigma_\gamma}\frac{d\sigma_\gamma}{dz} = \frac{d}{d\gamma}\left(\frac{d\gamma}{dz}\right),$$

and electronic stopping power of Bethe [6]

$$\frac{d\gamma}{dz} = -\frac{a_s L_s}{\bar{m}\beta^2}, \quad L_s = \ln(b_s\eta^2) - \beta^2, \tag{2}$$

we obtain the longitudinal cooling partition number

$$J_z = -2 + \frac{2[1 + \eta^2 \ln(b_s\eta^2)]}{\gamma^2 L_s},$$

where $a_s = 4\pi r_e^2 n_e$, $b_s = 2m_e c^2/I$, $\bar{m} = m/m_e$, m is the rest mass of beam particle, m_e and r_e are the rest mass and classical radius of electron, n_e is electron volume density and I is average ionization energy of the absorber.

The heating effects in an absorber include multiple scattering [6] which induces an angle spread

$$\sigma_{x's} = \frac{\sqrt{(1+Z)2L_b a_s h}}{\bar{m}\beta\eta}, \tag{3}$$

and straggling [6] which induces a momentum spread

$$\delta_{ps} = \frac{\sqrt{(1+\eta^2/2)a_s h}}{\bar{m}\beta\eta}, \tag{4}$$

where

$$L_b = \ln\left(\frac{183}{Z^{1/3}}\right),$$

which is related to radiation length by [6]

$$X_0 = \frac{\pi}{\alpha a_s (1+Z) L_b},$$

α is the fine structure constant, h is the thickness and Z is the atomic number of the absorber. Given initial and final momentum of the particle, the absorber thickness can be determined by

$$a_s h = \int_{\eta_f}^{\eta_i} \frac{\bar{m}\eta^3 d\eta}{\sqrt{1+\eta^2}[(1+\eta^2)\ln(b_s\eta^2) - \eta^2]}.$$

Assuming σ_x will not change significantly through the absorber, the transverse heating can be related to the angle spread induced by multiple scattering through [1, 5]

$$\frac{1}{\epsilon_x}\frac{d\epsilon_x}{dz} = \frac{\gamma\beta\beta_\perp}{2\epsilon_x}\frac{d\sigma_{x's}^2}{dz}, \tag{5}$$

where $\beta_\perp$ is the beta function. In case of a solenoid field, $\beta_\perp = \alpha_g \bar{m}\eta/B_s$, where $\alpha_g = 2m_e c/e$. From Eqs.(2,3,5), transverse heating partition number is

$$K_x = -\frac{\epsilon_0}{\epsilon_x}, \quad \epsilon_0 = \frac{\beta\beta_\perp(1+Z)L_b}{\bar{m}L_s}.$$

Similarly, the longitudinal heating partition number due to straggling can be derived from Eqs.(1,2,4)

$$K_z = -\frac{1+\gamma^2}{4\bar{m}\delta_p^2\gamma L_s}.$$

Summarizing all results on partition numbers gives

$$J_6 = \frac{2[1+\eta^2\ln(b_s\eta^2)]}{\gamma^2 L_s},$$

$$K_6 = -\frac{\epsilon_0}{\epsilon_x} - \frac{\epsilon_0}{\epsilon_y} - \frac{1+\gamma^2}{4\bar{m}\delta_p^2\gamma L_s},$$

$$J_6 + K_6 = J_6\left[1 - \left(\frac{\delta_0}{\delta_p}\right)^2\right] - \frac{\epsilon_0}{\epsilon_x} - \frac{\epsilon_0}{\epsilon_y},$$

$$\delta_0^2 = \frac{\gamma(1+\gamma^2)}{8\bar{m}[1+\eta^2\ln(b_s\eta^2)]}.$$

To maintain cooling in 6D, the minimum transverse emittance is constrained by $J_6 + K_6 = 0$, yielding

$$\frac{2\epsilon_0}{\epsilon_{min}} = J_6\left[1 - \left(\frac{\delta_0}{\delta_p}\right)^2\right],$$

and correspondingly

$$\sigma_{x'min} = \sqrt{\frac{\epsilon_{min}}{\eta\beta_\perp}}, \quad \sigma_{xmin} = \sqrt{\frac{\beta_\perp\epsilon_{min}}{\eta}}.$$

3 METHODOLOGY

The ultimate goal of a scaled experiment is to validate physics first without having to commit time and resource into technology development which may or may not be useful in the end depending on the verdict of physics validation. Driven by such a goal, our design philosophy is first to make the scaled experiment as convenient, flexible, and inexpensive as possible, and then to extract as much essential physics as possible through ingenious scaling and benchmark with theory and digital simulation.

An example is given in Table 1 for proton cooling with Be foil. For comparison, a typical case [1] is given in Table 2 for muon cooling with liquid H_2. In both cases, it is seen that cooling in 6D phase space is possible since $J_6 > 0$, but cooling in longitudinal phase space is impossible without emittance exchange since $J_z = J_6 - 2 < 0$ below minimum ionization. It is noted that we have used a much lower solenoid field and a much easier-to-handle absorber for proton cooling. As a result, both σ_{xmin} and $\sigma_{x'min}$ have larger values for proton beam. However, if we use the same solenoid field ($B_s = 15$T) and absorber (liquid H_2) for proton as for muon, we would have $\sigma_{xmin} = 2.5$mm and $\sigma_{x'min} = 76$mr for proton beam. In calculations, we have used $I = 64$eV, $n_e = 4.95 \times 10^{23}/\mathrm{cm}^3$ for Be, and $I = 22$eV, $n_e = 0.423 \times 10^{23}/\mathrm{cm}^3$ for liquid H_2.

Table 1: Example of Proton through Be

E_k (MeV)	3	δ_0 (%)	1.2
p (MeV/c)	75	δ_p (%)	5
η	0.08	ϵ_0 (mm-mr)	22
J_6	0.44	ϵ_{min}(mm-mr)	107
B_s (T)	5	σ_{xmin} (mm)	12
$\beta_\perp$ (cm)	10	$\sigma_{x'min}$ (mr)	115
$\Delta E_k/E_k$ (%)	20	$\sigma_{x's}$ (mr)	23
h (μm)	30	δ_{ps} (%)	0.47

Table 2: Example of Muon through Liquid H_2

E_k (MeV)	120	δ_0 (%)	1.3
p (MeV/c)	200	δ_p (%)	5
η	1.9	ϵ_0 (mm-mr)	351
J_6	1.7	ϵ_{min}(mm-mr)	439
B_s (T)	15	σ_{xmin} (mm)	4.6
$\beta_\perp$ (cm)	8.9	$\sigma_{x'min}$ (mr)	51
$\Delta E_k/E_k$ (%)	10	$\sigma_{x's}$ (mr)	17
h (cm)	38	δ_{ps} (%)	0.87

An important parameter for proton cooling is proton energy. To avoid severe beam loss through an absorber, proton energy should not be too high to cause excessive nuclear interaction [7, 8], or too low to induce significant charge exchange [7]. Residual effects of these proton-specific interactions can be removed or normalized, noting that angular and energy characteristics of these interactions are distinctly different from those caused by the intrinsic ionization process. In addition, requirements on re-acceleration and magnetic field strength are relaxed at lower proton energy. In the range of a few MeV, proton interaction with Be foil has been well studied [9].

The concept of "scaled experiment" should be understood and exploited to our full advantage in the broadest sense. In Table 1 and Table 2, scaling is applied over particle type and energy, absorber type, cooling rate, and relative position on ionization curve. To extend the concept further, one may even speculate scaling from cooling to heating or vice versa. As shown in Table 1, the equilibrium emittance is much larger than what can be produced with available proton sources. To demonstrate cooling, source emittance has to be increased first. This can be done easily through the same ionization process, for example, by placing an absorber foil in a high beta region. However, the behavior of heating, if well benchmarked with theory and digital simulation, should also tell us a lot about cooling.

An important advantage of analog simulation is that various difficult issues of beam dynamics can be studied over a wide range of scaled parameter space in a controlled fashion. For example, effects of non-paraxial beam and nonlinearity on emittance exchange can be studied gradually as proton emittance is increased from a small initial value due to heating, a convenient control knob not available with muons. In addition, space charge effects can be studied by varying proton current. Another important advantage of analog simulation is its flexibility as a toy machine, which can be quickly transformed and outfitted to test and optimize various different cooling techniques [10].

4 CONCLUSIONS

One day in Berkeley, I got a fortune cookie [11], it says: "If you have a difficult task, give it to a lazy man — he will find an easier way to do it". Enlightened, I hereby give it a try. This work was supported the U.S. Department of Energy under contract No.DE-AC03-76SF00098.

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